ON STRONGLY REGULAR GRAPHS WITH 
\( m_2 = qm_3 \) AND \( m_3 = qm_2 \) FOR \( q = 5, 6, 7, 8 \)

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Dedicated to my mother Jordana Lepović

ABSTRACT. We say that a regular graph \( G \) of order \( n \) and degree \( r \geq 1 \) (which is not the complete graph) is strongly regular if there exist non-negative integers \( \tau \) and \( \theta \) such that \( |S_i \cap S_j| = \tau \) for any two adjacent vertices \( i \) and \( j \), and \( |S_i \cap S_j| = \theta \) for any two distinct non-adjacent vertices \( i \) and \( j \), where \( S_k \) denotes the neighborhood of the vertex \( k \). Let \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) be the distinct eigenvalues of a connected strongly regular graph. Let \( m_1 = 1, m_2 \) and \( m_3 \) denote the multiplicity of \( r, \lambda_2, \) and \( \lambda_3 \), respectively. We describe the parameters \( n, r, \tau \) and \( \theta \) for strongly regular graphs with \( m_2 = qm_3 \) and \( m_3 = qm_2 \) for \( q = 5, 6, 7, 8 \).

1. INTRODUCTION

Let \( G \) be a simple graph of order \( n \) with vertex set \( V(G) = \{1, 2, \ldots, n\} \). The spectrum of \( G \) consists of the eigenvalues \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \) of its \((0,1)\) adjacency matrix \( A \) and is denoted by \( \sigma(G) \). We say that a regular graph \( G \) of order \( n \) and degree \( r \geq 1 \) (which is not the complete graph \( K_n \)) is strongly regular if there exist non-negative integers \( \tau \) and \( \theta \) such that \( |S_i \cap S_j| = \tau \) for any two adjacent vertices \( i \) and \( j \), and \( |S_i \cap S_j| = \theta \) for any two distinct non-adjacent vertices \( i \) and \( j \), where \( S_k \subseteq V(G) \) denotes the neighborhood of the vertex \( k \). We know that a regular connected graph \( G \) is strongly regular if and only if it has exactly three distinct eigenvalues [1] (see also [3]). Let \( \lambda_1 = r, \lambda_2, \) and \( \lambda_3 \) denote the distinct eigenvalues of a connected strongly regular graph \( G \). Let \( m_1 = 1, m_2 \) and \( m_3 \) denote the multiplicity of \( r, \lambda_2, \) and \( \lambda_3 \). Further, let \( \tau = (n - 1) - r, \lambda_2 = -\lambda_3 - 1 \) and \( \lambda_3 = -\lambda_2 - 1 \) denote the distinct eigenvalues of the strongly regular graph \( \overline{G} \), where \( \overline{G} \) denotes the complement of \( G \). Then \( \overline{\tau} = n - 2r - 2 + \theta \) and \( \overline{\theta} = n - 2r + \tau \), where \( \overline{\tau} = \tau(\overline{G}) \) and \( \overline{\theta} = \theta(\overline{G}) \).

Remark 1.1. (i) if \( G \) is a disconnected strongly regular graph of degree \( r \) then \( G = mK_{r+1} \), where \( mH \) denotes the \( m \)-fold union of the graph \( H \); (ii) \( G \) is a disconnected strongly regular graph if and only if \( \theta = 0 \).

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Remark 1.2. (i) a strongly regular graph $G$ of order $n = 4k + 1$ and degree $r = 2k$ with $\tau = k - 1$ and $\theta = k$ is called a conference graph; (ii) a strongly regular graph is a conference graph if and only if $m_2 = m_3$ and (iii) if $m_2 \neq m_3$ then $G$ is an integral\(^1\) graph.

We have recently started to investigate strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, where $q$ is a positive integer [4]. In particular, in the same work we have described the parameters $n, r, \tau$ and $\theta$ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 2, 3, 4$. We now proceed to establish the parameters of strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 5, 6, 7, 8$, as follows. Firstly,

**Proposition 1.1** (Elzinga [2]). Let $G$ be a connected or disconnected strongly regular graph of order $n$ and degree $r$. Then

$$r^2 - (\tau - \theta + 1)r - (n - 1)\theta = 0. \quad (1.1)$$

**Proposition 1.2** (Elzinga [2]). Let $G$ be a connected strongly regular graph of order $n$ and degree $r$. Then

$$2r + (\tau - \theta)(m_2 + m_3) + \delta(m_2 - m_3) = 0, \quad (1.2)$$

where $\delta = \lambda_2 - \lambda_3$.

Remark 1.3 (Lepović [4]). Using the same procedure applied in [4] we can establish the parameters $n, r, \tau$ and $\theta$ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for any fixed value $q \in \mathbb{N}$, as follows. Firstly, let $m_3 = p, m_2 = qp$ and $n = (q + 1)p + 1$, where $q \in \mathbb{N}$. Using (1.2) we obtain $r = p(\lambda_3 - q\lambda_2)$. Let $|\lambda_3| - q\lambda_2 = t$, where $t = 1, 2, \ldots, q$. Let $\lambda_2 = k$, where $k$ is a positive integer. Then (i) $\lambda_3 = -(qk + t)$; (ii) $\tau - \theta = -((q - 1)k + t)$; (iii) $\delta = (q + 1)k + t$; (iv) $r = pt$ and (v) $\theta = pt - qk^2 - kt$. Using (ii), (iv) and (v) we can easily see that (1.1) is reduced to

$$(p + 1)t^2 - ((q + 1)p + 1)t + q(q + 1)k^2 + 2qkt = 0. \quad (1.3)$$

Secondly, let $m_2 = p, m_3 = qp$ and $n = (q + 1)p + 1$, where $q \in \mathbb{N}$. Using (1.2) we obtain $r = p(q|\lambda_3| - \lambda_2)$. Let $q|\lambda_3| - \lambda_2 = t$, where $t = 1, 2, \ldots, q$. Let $\lambda_3 = -k$, where $k$ is a positive integer. Then (i) $\lambda_2 = qk - t$; (ii) $\tau - \theta = (q - 1)k - t$; (iii) $\delta = (q + 1)k - t$; (iv) $r = pt$ and (v) $\theta = pt - qk^2 + kt$. Using (ii), (iv) and (v) we can easily see that (1.1) is reduced to

$$(p + 1)t^2 - ((q + 1)p + 1)t + q(q + 1)k^2 - 2qkt = 0. \quad (1.4)$$

Using (1.3) and (1.4) we can obtain for $t = 1, 2, \ldots, q$ the corresponding classes of strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, respectively.

\(^1\)We say that a connected or disconnected graph $G$ is integral if its spectrum $\sigma(G)$ consists only of integral values.
2. Main results

Remark 2.1. We firstly describe the corresponding classes of connected strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ obtained in Propositions 2.1 and 2.2, respectively, then we prove Theorem 2.3 which is related to connected strongly regular graphs with $m_2 = qm_3$ or $m_3 = qm_2$, where $q = 5, 6, 7, 8$ and $k = 1, 2, 3, 4$.

Remark 2.2. Since $m_2(G) = m_3(G)$ and $m_3(G) = m_2(G)$ we note that if $m_2(G) = qm_3(G)$ then $m_3(G) = qm_2(G)$.

Remark 2.3. In Theorems 2.1, 2.2, 2.3 and 2.4 the complements of strongly regular graphs appear in pairs in $(k^0)$ and $(\overline{k}^0)$ classes, where $k$ denotes the corresponding number of a class.

Remark 2.4. $\overline{\alpha K_\beta}$ is a strongly regular graph of order $n = \alpha \beta$ and degree $r = (\alpha - 1)\beta$ with $\tau = (\alpha - 2)\beta$ and $\theta = (\alpha - 1)\beta$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -\beta$ with $m_2 = \alpha(\beta - 1)$ and $m_3 = \alpha - 1$.

Proposition 2.1. Let $G$ be a connected strongly regular graph of order $n$ and degree $r$ with $m_2 = 5m_3$. Then $G$ belongs to the class $(2^0)$ or $(3^0)$ represented in Theorem 2.1.

Proof. Let $m_3 = p$, $m_2 = 5p$ and $n = 6p + 1$, where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p|\lambda_3| - 5\lambda_2|$, where $t = 1, 2, \ldots, 5$. Let $\lambda_2 = k$, where $k$ is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_3 = -(5k + t)$; (ii) $\tau - \theta = -(4k + t)$; (iii) $\delta = 6k + t$; (iv) $r = pt$ and (v) $\theta = pt - 5k^2 - kt$. In this case we can easily see that (1.3) is reduced to

\[(p + 1)^2 - (6p + 1)t + 30k^2 + 10kt = 0.\]  \hspace{1cm} (2.1)

Case 1. ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(5k + 1)$, $\tau - \theta = -(4k + 1)$, $\delta = 6k + 1$, $r = p$ and $\theta = p - 5k^2 - k$. Using (2.1) we find that $4p - 1 = 5k(3k + 1)$, a contradiction because $4 \nmid 15k^2 + 10k + 1$.

Case 2. ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(5k + 1)$, $\tau - \theta = -(4k + 2)$, $\delta = 6k + 2$, $r = 2p$ and $\theta = 2p - 5k^2 - 2k$. Using (2.1) we find that $4p - 2 = 10k(k + 1)$, a contradiction because $4 \nmid 10k^2 + 10k + 2$.

Case 3. ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(5k + 3)$, $\tau - \theta = -(4k + 3)$, $\delta = 6k + 3$, $r = 3p$ and $\theta = 3p - 5k^2 - 3k$. Using (2.1) we find that $4p - 2 = 10k(k + 1)$, a contradiction because $4 \nmid 10k^2 + 10k + 2$.

Case 4. ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(5k + 4)$, $\tau - \theta = -(4k + 4)$, $\delta = 6k + 4$, $r = 4p$ and $\theta = 4p - 5k^2 - 4k$. Using (2.1) we find that $4p - 6 = 5k(3k + 4)$, a contradiction because $4 \nmid 15k^2 + 20k + 6$.

Case 5. ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(5k + 5)$, $\tau - \theta = -(4k + 5)$, $\delta = 6k + 5$, $r = 5p$ and $\theta = 5p - 5k^2 - 5k$. Using
we find that \( p = 2(k + 1)(3k + 2) \). Replacing \( k \) with \( k - 1 \) we arrive at \( p = 2k(3k - 1) \). So we obtain that \( G \) is a strongly regular graph of order \( n = (6k - 1)^2 \) and degree \( r = 10k(3k - 1) \) with \( \tau = 25k^2 - 9k - 1 \) and \( \theta = 5k(5k - 1) \).

**Proposition 2.2.** Let \( G \) be a connected strongly regular graph of order \( n \) and degree \( r \) with \( m_3 = 5m_2 \). Then \( G \) belongs to the class \((2^0)\) or \((3^0)\) represented in Theorem 2.1.

**Proof.** Let \( m_2 = p, m_3 = 5p \) and \( n = 6p + 1 \), where \( p \in \mathbb{N} \). Using (1.2) we obtain \( r = p(5|\lambda_3| - \lambda_2) \). Let \( 5|\lambda_3| - \lambda_2 = t \), where \( t = 1, 2, \ldots, 5 \). Let \( \lambda_3 = -k \), where \( k \) is a positive integer. Then according to Remark 1.3 we have (i) \( \lambda_2 = 5k - t \); (ii) \( \tau = 4k - t \); (iii) \( \delta = 6k - t \); (iv) \( r = pt \) and (v) \( \theta = pt - 5k^2 + kt \). In this case we can easily see that (1.4) is reduced to

\[
(p + 1)t^2 - (6p + 1)t + 30k^2 - 10kt = 0. \tag{2.2}
\]

**Case 1.** \((t = 1)\). Using (i), (ii), (iii), (iv) and (v) we find that \( \lambda_2 = 5k - 1 \) and \( \lambda_3 = -k \), \( \tau = \theta = 4k - 1 \), \( \delta = 6k - 1 \), \( r = p \) and \( \theta = p - 5k^2 + k \). Using (2.2) we find that \( p = 2k(3k - 1) \). So we obtain that \( G \) is a strongly regular graph of order \( n = (6k - 1)^2 \) and degree \( r = 2k(3k - 1) \) with \( \tau = k^2 + 3k - 1 \) and \( \theta = k(k - 1) \).

**Case 2.** \((t = 2)\). Using (i), (ii), (iii), (iv) and (v) we find that \( \lambda_2 = 5k - 2 \) and \( \lambda_3 = -k \), \( \tau = \theta = 4k - 2 \), \( \delta = 6k - 2 \), \( r = 2p \) and \( \theta = 2p - 5k^2 + 2k \). Using (2.2) we find that \( 4p - 1 = 5k(3k - 2) \), a contradiction because \( 4 \nmid 15k^2 - 10k + 1 \).

**Case 3.** \((t = 3)\). Using (i), (ii), (iii), (iv) and (v) we find that \( \lambda_2 = 5k - 3 \) and \( \lambda_3 = -k \), \( \tau = \theta = 4k - 3 \), \( \delta = 6k - 3 \), \( r = 3p \) and \( \theta = 3p - 5k^2 + 3k \). Using (2.2) we find that \( 3p - 2 = 10k(k - 1) \), a contradiction because \( 3 \nmid 10k^2 - 10k + 2 \).

**Case 4.** \((t = 4)\). Using (i), (ii), (iii), (iv) and (v) we find that \( \lambda_2 = 5k - 4 \) and \( \lambda_3 = -k \), \( \tau = \theta = 4k - 4 \), \( \delta = 6k - 4 \), \( r = 4p \) and \( \theta = 4p - 5k^2 + 4k \). Using (2.2) we find that \( 4p - 6 = 5k(3k - 4) \), a contradiction because \( 4 \nmid 15k^2 - 20k + 6 \).

**Case 5.** \((t = 5)\). Using (i), (ii), (iii), (iv) and (v) we find that \( \lambda_2 = 5k - 5 \) and \( \lambda_3 = -k \), \( \tau = \theta = 4k - 5 \), \( \delta = 6k - 5 \), \( r = 5p \) and \( \theta = 5p - 5k^2 + 5k \). Using (2.2) we find that \( p = 2(k - 1)(3k - 2) \). Replacing \( k \) with \( k + 1 \) we arrive at \( p = 2k((3k + 1) \). So we obtain that \( G \) is a strongly regular graph of order \( n = (6k + 1)^2 \) and degree \( r = 10k(3k + 1) \) with \( \tau = 25k^2 + 9k - 1 \) and \( \theta = 5k(5k + 1) \).

**Remark 2.5.** We note that \( \overline{5K^5} \) is a strongly regular graph with \( m_2 = 5m_3 \). It is obtained from the class Theorem 2.1 \((2^0)\) for \( k = 1 \).

**Theorem 2.1.** Let \( G \) be a connected strongly regular graph of order \( n \) and degree \( r \) with \( m_2 = 5m_3 \) or \( m_3 = 5m_2 \). Then \( G \) is one of the following strongly regular graphs:

\[
(1^0) \ G \ is \ the \ strongly \ regular \ graph \ \overline{5K^5} \ of \ order \ n = 25 \ and \ degree \ r = 20 \ with \ \tau = 15 \ and \ \theta = 20. \ \ Its \ eigenvalues \ are \ \lambda_2 = 0 \ and \ \lambda_3 = -5 \ with \ m_2 = 20 \ and \ m_3 = 4. ;
\]
\( (2^0) \) \( G \) is a strongly regular graph of order \( n = (6k-1)^2 \) and degree \( r = 2k(3k-1) \) with \( \tau = k^2 + 3k - 1 \) and \( \theta = k(k-1) \), where \( k \geq 2 \). Its eigenvalues are \( \lambda_2 = 5k - 1 \) and \( \lambda_3 = -k \) with \( m_2 = 2k(3k-1) \) and \( m_3 = 10k(3k-1) \);

\( (2^1) \) \( G \) is a strongly regular graph of order \( n = (6k-1)^2 \) and degree \( r = 10k(3k-1) \) with \( \tau = 25k^2 - 9k - 1 \) and \( \theta = 5k(5k-1) \), where \( k \geq 2 \). Its eigenvalues are \( \lambda_2 = k - 1 \) and \( \lambda_3 = -5k \) with \( m_2 = 10k(3k-1) \) and \( m_3 = 2k(3k-1) \);

\( (3^0) \) \( G \) is a strongly regular graph of order \( n = (6k+1)^2 \) and degree \( r = 2k(3k+1) \) with \( \tau = k^2 - 3k - 1 \) and \( \theta = k(k+1) \), where \( k \geq 4 \). Its eigenvalues are \( \lambda_2 = k \) and \( \lambda_3 = -(5k+1) \) with \( m_2 = 10k(3k+1) \) and \( m_3 = 2k(3k+1) \);

\( (3^1) \) \( G \) is a strongly regular graph of order \( n = (6k+1)^2 \) and degree \( r = 10k(3k+1) \) with \( \tau = 25k^2 + 9k - 1 \) and \( \theta = 5k(5k+1) \), where \( k \geq 4 \). Its eigenvalues are \( \lambda_2 = 5k \) and \( \lambda_3 = -(k+1) \) with \( m_2 = 2k(3k+1) \) and \( m_3 = 10k(3k+1) \).

**Proof.** Firstly, according to Remark 2.4 we have \( \alpha(\beta - 1) = 5(\alpha - 1) \), from which we find that \( \alpha = 5, \beta = 5 \). In view of this we obtain the strongly regular graph represented in Theorem 2.1 \( (1^0) \). Next, according to Proposition 2.1 it turns out that \( G \) belongs to the class \( (2^0) \) or \( (3^0) \) if \( m_2 = 5m_3 \). According to Proposition 2.2 it turns out that \( G \) belongs to the class \( (2^0) \) or \( (3^0) \) if \( m_3 = 5m_2 \). \( \square \)

**Proposition 2.3.** Let \( G \) be a connected strongly regular graph of order \( n \) and degree \( r \) with \( m_2 = 6m_3 \). Then \( G \) belongs to the class \( (2^0) \) or \( (5^0) \) or \( (6^0) \) or \( (7^0) \) or \( (8^0) \) or \( (9^0) \) represented in Theorem 2.2.

**Proof.** Let \( m_3 = p, m_2 = 6p \) and \( n = 7p + 1 \), where \( p \in \mathbb{N} \). Using (1.2) we obtain \( r = p(\lambda_3 - 6\lambda_2) \). Let \( \lambda_3 - 6\lambda_2 = t \), where \( t = 1, 2, \ldots, 6 \). Let \( \lambda_2 = k \), where \( k \) is a positive integer. Then according to Remark 1.3 we have (i) \( \lambda_3 = -(6k+1) \); (ii) \( \tau - \theta = -(5k+1) \); (iii) \( \delta = 7k + t \); (iv) \( r = pt \) and (v) \( \theta = pt - 6k^2 - kt \). In this case we can easily see that (1.3) is reduced to

\[
(p + 1)^2 - (7p + 1)t + 42k^2 + 12kt = 0. \tag{2.3}
\]

**Case 1.** \( (t = 1) \). Using (i), (ii), (iii), (iv) and (v) we find that \( \lambda_2 = k \) and \( \lambda_3 = -(6k + 1), \tau - \theta = -(5k + 1), \delta = 7k + 1, r = p \) and \( \theta = p - 6k^2 - k \). Using (2.3) we find that \( p = k(7k + 2) \). So we obtain that \( G \) is a strongly regular graph of order \( n = (7k + 1)^2 \) and degree \( r = k(7k + 2) \) with \( \tau = k^2 - 4k - 1 \) and \( \theta = k(k+1) \).

**Case 2.** \( (t = 2) \). Using (i), (ii), (iii), (iv) and (v) we find that \( \lambda_2 = k \) and \( \lambda_3 = -(6k + 2), \tau - \theta = -(5k + 2), \delta = 7k + 2, r = 2p \) and \( \theta = 2p - 6k^2 - 2k \). Using (2.3) we find that \( 5p - 1 = 3k(7k + 4) \). Replacing \( k \) with \( 5k - 1 \) we arrive at \( p = 105k^2 - 30k + 2 \). So we obtain that \( G \) is a strongly regular graph of order \( n = 15(7k - 1)^2 \) and degree \( r = 2(105k^2 - 30k + 2) \) with \( \tau = 60k^2 - 35k + 3 \) and \( \theta = 10k(6k - 1) \).

**Case 3.** \( (t = 3) \). Using (i), (ii), (iii), (iv) and (v) we find that \( \lambda_2 = k \) and \( \lambda_3 = -(6k + 3), \tau - \theta = -(5k + 3), \delta = 7k + 3, r = 3p \) and \( \theta = 3p - 6k^2 - 3k \). Using
(2.3) we find that $2p - 1 = k(7k + 6)$. Replacing $k$ with $2k - 1$ we arrive at $p = 14k^2 - 8k + 1$. So we obtain that $G$ is a strongly regular graph of order $n = 2(7k - 2)^2$ and degree $r = 3(14k^2 - 8k + 1)$ with $\tau = 18k^2 - 16k + 2$ and $\theta = 6k(3k - 1)$.

**Case 4.** $(t = 4)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(6k + 4)$, $\tau - \theta = -(5k + 4)$, $\delta = 7k + 4$, $r = 4p$ and $\theta = 4p - 6k^2 - 4k$. Using (2.3) we find that $2p - 2 = k(7k + 8)$. Replacing $k$ with $2k$ we arrive at $p = 14k^2 + 8k + 1$. So we obtain that $G$ is a strongly regular graph of order $n = 2(7k + 2)^2$ and degree $r = 4(14k^2 + 8k + 1)$ with $\tau = 2k(16k + 7)$ and $\theta = 4(2k + 1)(4k + 1)$.

**Case 5.** $(t = 5)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(6k + 5)$, $\tau - \theta = -(5k + 5)$, $\delta = 7k + 5$, $r = 5p$ and $\theta = 5p - 6k^2 - 5k$. Using (2.3) we find that $5p - 10 = 3k(7k + 10)$. Replacing $k$ with $5k$ we arrive at $p = 105k^2 + 30k + 2$. So we obtain that $G$ is a strongly regular graph of order $n = 15(7k + 1)^2$ and degree $r = 5(105k^2 + 30k + 2)$ with $\tau = 5(5k + 1)(15k + 1)$ and $\theta = 5(5k + 1)(15 + 2)$.

**Case 6.** $(t = 6)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(6k + 6)$, $\tau - \theta = -(5k + 6)$, $\delta = 7k + 6$, $r = 6p$ and $\theta = 6p - 6k^2 - 6k$. Using (2.3) we find that $p = (k + 1)(7k + 5)$. Replacing $k$ with $k - 1$ we arrive at $p = k(7k - 2)$. So we obtain that $G$ is a strongly regular graph of order $n = (7k - 1)^2$ and degree $r = 6k(7k - 2)$ with $\tau = 36k^2 - 11k - 1$ and $\theta = 6k(6k - 1)$.

**Proposition 2.4.** Let $G$ be a connected strongly regular graph of order $n$ and degree $r$ with $m_3 = 6m_2$. Then $G$ belongs to the class (4$^0$) or (5$^0$) or (6$^0$) or (7$^0$) or (8$^0$) or (9$^0$) represented in Theorem 2.2.

**Proof.** Let $m_2 = p$, $m_3 = 6p$ and $n = 7p + 1$, where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(6|\lambda_3| - \lambda_2)$. Let $6|\lambda_3| - \lambda_2 = t$, where $t = 1, 2, \ldots, 6$. Let $\lambda_3 = -k$, where $k$ is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_2 = 6k - t$; (ii) $\tau - \theta = 5k - t$; (iii) $\delta = 7k - t$; (iv) $r = pt$ and (v) $\theta = pt - 6k^2 + kt$. In this case we can easily see that (1.4) is reduced to

$$(p + 1)^2 - (7p + 1)t + 42k^2 - 12kt = 0. \quad (2.4)$$

**Case 1.** $(t = 1)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 6k - 1$ and $\lambda_3 = -k$, $\tau - \theta = 5k - 1$, $\delta = 7k - 1$, $r = p$ and $\theta = p - 6k^2 + k$. Using (2.4) we find that $p = k(7k - 2)$. So we obtain that $G$ is a strongly regular graph of order $n = (7k - 1)^2$ and degree $r = k(7k - 2)$ with $\tau = k^2 + 4k - 1$ and $\theta = k(k - 1)$.

**Case 2.** $(t = 2)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 6k - 2$ and $\lambda_3 = -k$, $\tau - \theta = 5k - 2$, $\delta = 7k - 2$, $r = 2p$ and $\theta = 2p - 6k^2 + 2k$. Using (2.4) we find that $5p - 1 = 3k(7k - 4)$. Replacing $k$ with $5k + 1$ we arrive at $p = 105k^2 + 30k + 2$. So we obtain that $G$ is a strongly regular graph of order $n = 15(7k + 1)^2$ and degree $r = 2(105k^2 + 30k + 2)$ with $\tau = 60k^2 + 35k + 3$ and $\theta = 10k(6k + 1)$.

**Case 3.** $(t = 3)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 6k - 3$ and $\lambda_3 = -k$, $\tau - \theta = 5k - 3$, $\delta = 7k - 3$, $r = 3p$ and $\theta = 3p - 6k^2 + 3k$. Using (2.4) we
find that $2p - 1 = k(7k - 6)$. Replacing $k$ with $2k + 1$ we arrive at $p = 14k^2 + 8k + 1$. So we obtain that $G$ is a strongly regular graph of order $n = 2(7k + 2)^2$ and degree $r = 3(14k^2 + 8k + 1)$ with $\tau = 18k^2 + 16k + 2$ and $\theta = 6k(3k + 1)$.

**Case 4.** ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 6k - 4$ and $\lambda_3 = -k$, $\tau - \theta = 5k - 4$, $\delta = 7k - 4$, $r = 4p$ and $\theta = 4p - 6k^2 + 4k$. Using (2.4) we find that $2p - 2 = k(7k - 8)$. Replacing $k$ with $2k$ we arrive at $p = 14k^2 - 8k + 1$. So we obtain that $G$ is a strongly regular graph of order $n = 2(7k - 2)^2$ and degree $r = 4(14k^2 - 8k + 1)$ with $\tau = 2k(16k - 7)$ and $\theta = 4(2k - 1)(4k - 1)$.

**Case 5.** ($t = 5$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 6k - 5$ and $\lambda_3 = -k$, $\tau - \theta = 5k - 5$, $\delta = 7k - 5$, $r = 5p$ and $\theta = 5p - 6k^2 + 5k$. Using (2.4) we find that $5p - 10 = 3k(7k - 10)$. Replacing $k$ with $5k$ we arrive at $p = 105k^2 - 30k + 2$. So we obtain that $G$ is a strongly regular graph of order $n = 15(7k - 1)^2$ and degree $r = 5(105k^2 - 30k + 2)$ with $\tau = 5(5k - 1)(15k - 1)$ and $\theta = 5(5k - 1)(15k - 2)$.

**Case 6.** ($t = 6$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 6k - 6$ and $\lambda_3 = -k$, $\tau - \theta = 5k - 6$, $\delta = 7k - 6$, $r = 6p$ and $\theta = 6p - 6k^2 + 6k$. Using (2.4) we find that $p = (k - 1)(7k - 5)$. Replacing $k$ with $k + 1$ we arrive at $p = k(7k + 2)$. So we obtain that $G$ is a strongly regular graph of order $n = (7k + 1)^2$ and degree $r = 6k(7k + 2)$ with $\tau = 36k^2 + 11k - 1$ and $\theta = 6k(6k + 1)$.

**Remark 2.6.** We note that the complete bipartite graph $K_{4,4}$ is a strongly regular graph with $m_2 = 6m_3$. It is obtained from the class Theorem 2.2 ($\mathcal{T}^0$) for $k = 0$.

**Remark 2.7.** We note that $\overline{3K_5}$ is a strongly regular graph with $m_2 = 6m_3$. It is obtained from the class Theorem 2.2 ($\mathcal{T}^0$) for $k = 0$.

**Remark 2.8.** We note that $\overline{6K_6}$ is a strongly regular graph with $m_2 = 6m_3$. It is obtained from the class Theorem 2.2 ($\mathcal{T}^0$) for $k = 1$.

**Theorem 2.2.** Let $G$ be a connected strongly regular graph of order $n$ and degree $r$ with $m_2 = 6m_3$ or $m_3 = 6m_2$. Then $G$ is one of the following strongly regular graphs:

1. $G$ is the complete bipartite graph $K_{4,4}$ of order $n = 8$ and degree $r = 4$ with $\tau = 0$ and $\theta = 4$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -4$ with $m_2 = 6$ and $m_3 = 1$;

2. $G$ is the strongly regular graph $\overline{3K_5}$ of order $n = 15$ and degree $r = 10$ with $\tau = 5$ and $\theta = 10$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -5$ with $m_2 = 12$ and $m_3 = 2$;

3. $G$ is the strongly regular graph $\overline{6K_6}$ of order $n = 36$ and degree $r = 30$ with $\tau = 24$ and $\theta = 30$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -6$ with $m_2 = 30$ and $m_3 = 5$;

4. $G$ is a strongly regular graph of order $n = (7k - 1)^2$ and degree $r = k(7k - 2)$ with $\tau = k^2 + 4k - 1$ and $\theta = k(k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 6k - 1$ and $\lambda_3 = -k$ with $m_2 = k(7k - 2)$ and $m_3 = 6k(7k - 2)$;
(4) $G$ is a strongly regular graph of order $n = (7k−1)^2$ and degree $r = 6k(7k−2)$ with $\tau = 36k^2−11k−1$ and $\theta = 6k(6k−1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k−1$ and $\lambda_3 = −6k$ with $m_2 = 6k(7k−2)$ and $m_3 = k(7k−2)$;

(5) $G$ is a strongly regular graph of order $n = (7k+1)^2$ and degree $r = k(7k+2)$ with $\tau = k^2−4k−1$ and $\theta = k(k+1)$, where $k \geq 5$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = −(6k+1)$ with $m_2 = 6k(7k+2)$ and $m_3 = k(7k+2)$;

(6) $G$ is a strongly regular graph of order $n = (7k+1)^2$ and degree $r = 6k(7k+2)$ with $\tau = 36k^2+11k−1$ and $\theta = 6k(6k+1)$, where $k \geq 5$. Its eigenvalues are $\lambda_2 = 6k$ and $\lambda_3 = −(k+1)$ with $m_2 = k(7k+2)$ and $m_3 = 6k(7k+2)$;

(7) $G$ is a strongly regular graph of order $n = 2(7k−2)^2$ and degree $r = 3(14k^2−8k+1)$ with $\tau = 18k^2−16k+2$ and $\theta = 6k(3k−1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k−4$ and $\lambda_3 = −2k$ with $m_2 = 14k^2−8k+1$ and $m_3 = 6(14k^2−8k+1)$;

(8) $G$ is a strongly regular graph of order $n = 2(7k+2)^2$ and degree $r = 3(14k^2+8k+1)$ with $\tau = 18k^2+16k+2$ and $\theta = 6k(3k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k+3$ and $\lambda_3 = −(2k+1)$ with $m_2 = 14k^2+8k+1$ and $m_3 = 6(14k^2+8k+1)$;

(9) $G$ is a strongly regular graph of order $n = 2(7k+2)^2$ and degree $r = 4(14k^2+8k+1)$ with $\tau = 2k(16k−7)$ and $\theta = 4(2k−1)(4k−1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k$ and $\lambda_3 = −(12k+4)$ with $m_2 = 6(14k^2+8k+1)$ and $m_3 = 14k^2+8k+1$;

(10) $G$ is a strongly regular graph of order $n = 15(7k−1)^2$ and degree $r = 2(105k^2−30k+2)$ with $\tau = 60k^2−35k+3$ and $\theta = 10k(6k−1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k−1$ and $\lambda_3 = −(30k−4)$ with $m_2 = 6(105k^2−30k+2)$ and $m_3 = 105k^2−30k+2$;

(11) $G$ is a strongly regular graph of order $n = 15(7k−1)^2$ and degree $r = 5(105k^2−30k+2)$ with $\tau = 5(5k−1)(15k−1)$ and $\theta = 5(5k−1)(15k−2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 30k−5$ and $\lambda_3 = −5k$ with $m_2 = 105k^2−30k+2$ and $m_3 = 6(105k^2−30k+2)$;

(12) $G$ is a strongly regular graph of order $n = 15(7k+1)^2$ and degree $r = 2(105k^2+30k+2)$ with $\tau = 60k^2+35k+3$ and $\theta = 10k(6k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 30k+4$ and $\lambda_3 = −(5k+1)$ with $m_2 = 105k^2+30k+2$ and $m_3 = 6(105k^2+30k+2)$;

(13) $G$ is a strongly regular graph of order $n = 15(7k+1)^2$ and degree $r = 5(105k^2+30k+2)$ with $\tau = 5(5k+1)(15k+1)$ and $\theta = 5(5k+1)(15+2)$,
Proposition 2.5. Let $G$ be a connected strongly regular graph of order $n$ and degree $r=\lambda_2=5k$ and $\lambda_3=-(30k+5)$ with $m_2=6(105k^2+30k+2)$ and $m_3=105k^2+30k+2$.

Proof. Firstly, according to Remark 2.4 we have $\alpha(\beta-1)=6(\alpha-1)$, from which we find that $\alpha=2, \beta=4$ or $\alpha=3, \beta=5$ or $\alpha=6, \beta=6$. In view of this we obtain the strongly regular graphs represented in Theorem 2.2 $(1^0), (2^0), (3^0)$. Next, according to Proposition 2.3 it turns out that $G$ belongs to the class $(4^0)$ or $(5^0)$ or $(6^0)$ or $(7^0)$ or $(8^0)$ or $(9^0)$ if $m_2=6m_2$. According to Proposition 2.4 it turns out that $G$ belongs to the class $(4^0)$ or $(5^0)$ or $(6^0)$ or $(7^0)$ or $(8^0)$ or $(9^0)$ if $m_3=6m_2$.

Case 5. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2=k$ and $\lambda_3=-(7k+1), \tau-\theta=-(6k+1), \delta=8k+1, r=p$ and $\theta=p-7k^2-kt$. In this case we can easily see that (1.3) is reduced to

$$(p+1)t^2-(8p+1)t+56k^2+14kt=0.$$  \hspace{1cm} (2.5)

Case 1. $(t=1)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2=k$ and $\lambda_3=-(7k+1), \tau-\theta=-(6k+1), \delta=8k+1, r=p$ and $\theta=p-7k^2-kt$. Using (2.5) we find that $p=2k(4k+1)$. So we obtain that $G$ is a strongly regular graph of order $n=(8k+1)^2$ and degree $r=2k(4k+1)$ with $\tau=k^2-5k-1$ and $\theta=k(k+1)$.

Case 2. $(t=2)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2=k$ and $\lambda_3=-(7k+2), \tau-\theta=-(6k+2), \delta=8k+2, r=2p$ and $\theta=2p-7k^2-2k$. Using (2.5) we find that $\delta=6p-1=14k(2k+1)$, a contradiction because $2 \nmid 6p-1$.

Case 3. $(t=3)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2=k$ and $\lambda_3=-(7k+3), \tau-\theta=-(6k+3), \delta=8k+3, r=3p$ and $\theta=3p-7k^2-3k$. Using (2.5) we find that $15p-6=14k(4k+3)$. Replacing $k$ with $15k-6$ we arrive at $p=840k^2-630k+118$. So we obtain that $G$ is a strongly regular graph of order $n=105(8k-3)^2$ and degree $r=3(840k^2-630k+118)$ with $\tau=945k^2-765k+153$ and $\theta=15(3k-1)(21k-8)$.

Case 4. $(t=4)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2=k$ and $\lambda_3=-(7k+4), \tau-\theta=-(6k+4), \delta=8k+4, r=4p$ and $\theta=4p-7k^2-4k$. Using (2.5) we find that $4p-3=14k(k+1)$, a contradiction because $2 \nmid 4p-3$.

Case 5. $(t=5)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2=k$ and $\lambda_3=-(7k+5), \tau-\theta=-(6k+5), \delta=8k+5, r=5p$ and $\theta=5p-7k^2-5k$. Using (2.5) we find that $15p-20=14k(4k+5)$. Replacing $k$ with $15k+5$ we arrive at $p=840k^2+630k+118$. So we obtain that $G$ is a strongly regular graph of order
Case 7. Let $G$ be a connected strongly regular graph of order $n$ and degree $r = 14k(4k-1)$ with $\tau = 49k^2 - 13k - 1$ and $\theta = 7k(7k-1)$. □

**Proposition 2.6.** Let $G$ be a connected strongly regular graph of order $n$ and degree $r$ with $m_3 = 7m_2$. Then $G$ belongs to the class $(2^0)$ or $(3^0)$ or $(4^0)$ or $(5^0)$ represented in Theorem 2.3.

**Proof.** Let $m_2 = p$, $m_3 = 7p$ and $n = 8p + 1$, where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(7\lambda_3 - \lambda_2)$. Let $7\lambda_3 - \lambda_2 = t$, where $t = 1, 2, \ldots, 7$. Let $\lambda_3 = -k$, where $k$ is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_2 = 7k - t$; (ii) $\tau - \theta = 6k - t$; (iii) $\delta = 8k - t$; (iv) $\tau = pt$ and (v) $\theta = pt - 7k^2 + kt$. In this case we can easily see that (1.4) is reduced to

$$(p+1)t^2 - (8p+1)t + 56k^2 - 14kt = 0. \tag{2.6}$$

**Case 1.** $(t = 1)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 7k - 1$ and $\lambda_3 = -k$, $\tau - \theta = 6k - 1$, $\delta = 8k - 1$, $r = p$ and $\theta = p - 7k^2 + k$. Using (2.6) we find that $p = 2k(4k-1)$. So we obtain that $G$ is a strongly regular graph of order $n = (8k-1)^2$ and degree $r = 2k(4k-1)$ with $\tau = k^2 + 5k - 1$ and $\theta = k(k-1)$.

**Case 2.** $(t = 2)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 7k - 2$ and $\lambda_3 = -k$, $\tau - \theta = 6k - 2$, $\delta = 8k - 2$, $r = 2p$ and $\theta = 2p - 7k^2 + 2k$. Using (2.6) we find that $6p - 1 = 14k(2k-1)$, a contradiction because $2 \nmid 6p - 1$.

**Case 3.** $(t = 3)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 7k - 3$ and $\lambda_3 = -k$, $\tau - \theta = 6k - 3$, $\delta = 8k - 3$, $r = 3p$ and $\theta = 3p - 7k^2 + 3k$. Using (2.6) we find that $15p - 6 = 14k(4k - 3)$. Replacing $k$ with $15k+6$ we arrive at $p = 840k^2 + 630k + 118$. So we obtain that $G$ is a strongly regular graph of order $n = 105(8k+3)^2$ and degree $r = 3(840k^2 + 630k + 118)$ with $\tau = 945k^2 + 765k + 153$ and $\theta = 15(3k+1)(21k+8)$.

**Case 4.** $(t = 4)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 7k - 4$ and $\lambda_3 = -k$, $\tau - \theta = 6k - 4$, $\delta = 8k - 4$, $r = 4p$ and $\theta = 4p - 7k^2 + 4k$. Using (2.6) we find that $4p - 3 = 14k(k-1)$, a contradiction because $2 \nmid 4p - 3$.

**Case 5.** $(t = 5)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 7k - 5$ and $\lambda_3 = -k$, $\tau - \theta = 6k - 5$, $\delta = 8k - 5$, $r = 5p$ and $\theta = 5p - 7k^2 + 5k$. Using (2.6) we find that $15p - 20 = 14k(4k - 5)$. Replacing $k$ with $15k+5$ we arrive at $p = 840k^2 - 630k + 118$. So we obtain that $G$ is a strongly regular graph of order $n = 105(8k+3)^2$ and degree $r = 5(840k^2 - 630k + 118)$ with $\tau = 5(525k^2 + 387k + 71)$ and $\theta = 15(5k+2)(35k+13)$. 

Case 6. $(t = 6)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(7k + 6)$, $\tau - \theta = -(6k + 6)$, $\delta = 8k + 6$, $r = 6p$ and $\theta = 6p - 7k^2 - 6k$. Using (2.5) we find that $6p - 15 = 14k(2k - 3)$, a contradiction because $2 \nmid 6p - 15$. 

Case 7. $(t = 7)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(7k + 7)$, $\tau - \theta = -(6k + 7)$, $\delta = 8k + 7$, $r = 7p$ and $\theta = 7p - 7k^2 - 7k$. Using (2.5) we find that $p = 2(k+1)(4k+3)$. Replacing $k$ with $k - 1$ we arrive at $p = 2k(4k - 1)$. So we obtain that $G$ is a strongly regular graph of order $n = (8k-1)^2$ and degree $r = 14k(4k-1)$ with $\tau = 49k^2 - 13k - 1$ and $\theta = 7k(7k-1)$.
105(8k−3)^2 and degree r = 5(840k^2 − 630k + 118) with τ = 5(525k^2 − 387k + 71) and θ = 15(5k−2)(35k−13).

Case 6. (t = 6). Using (i), (ii), (iii), (iv) and (v) we find that λ_2 = 7k−6 and λ_3 = −k, τ = 6k−6, δ = 8k−6, r = 6p and θ = 6p−7k^2+6k. Using (2.6) we find that 6p−15 = 14k(2k−3), a contradiction because 2∤6p−15.

Case 7. (t = 7). Using (i), (ii), (iii), (iv) and (v) we find that λ_2 = 7k−7 and λ_3 = −k, τ = 6k−7, δ = 8k−7, r = 7p and θ = 7p−7k^2+7k. Using (2.6) we find that p = 2(k−1)(4k−3). Replacing k with k + 1 we arrive at p = 2k(4k+1). So we obtain that G is a strongly regular graph of order n = (8k+1)^2 and degree r = 14k(4k+1) with τ = 49k^2+13k−1 and θ = 7k(7k+1).

Remark 2.9. We note that $\overline{7K_7}$ is a strongly regular graph with $m_2 = 7m_3$. It is obtained from the class Theorem 2.3 (2°) for $k = 1$.

Theorem 2.3. Let G be a connected strongly regular graph of order n and degree r with $m_2 = 7m_3$ or $m_3 = 7m_2$. Then G is one of the following strongly regular graphs:

(1°) G is the strongly regular graph $\overline{7K_7}$ of order n = 49 and degree r = 42 with τ = 35 and θ = 42. Its eigenvalues are λ_2 = 0 and λ_3 = −7 with $m_2 = 42$ and $m_3 = 6$;

(2°) G is a strongly regular graph of order n = (8k−1)^2 and degree r = 2k(4k−1) with τ = k^2 + 5k−1 and θ = k(k−1), where k ≥ 2. Its eigenvalues are λ_2 = 7k−1 and λ_3 = −k with $m_2 = 2k(4k−1)$ and $m_3 = 14k(4k−1)$;

(3°) G is a strongly regular graph of order n = (8k−1)^2 and degree r = 14k(4k−1) with τ = 49k^2−13k−1 and θ = 7k(7k−1), where k ≥ 2. Its eigenvalues are λ_2 = k−1 and λ_3 = −7k with $m_2 = 14k(4k−1)$ and $m_3 = 2k(4k−1)$;

(4°) G is a strongly regular graph of order n = (8k+1)^2 and degree r = 2k(4k+1) with τ = k^2−5k−1 and θ = k(k+1), where k ≥ 6. Its eigenvalues are λ_2 = k and λ_3 = −(7k+1) with $m_2 = 14k(4k+1)$ and $m_3 = 2k(4k+1)$;

(5°) G is a strongly regular graph of order n = (8k+1)^2 and degree r = 14k(4k+1) with τ = 49k^2+13k−1 and θ = 7k(7k+1), where k ≥ 6. Its eigenvalues are λ_2 = 7k and λ_3 = −(k+1) with $m_2 = 2k(4k+1)$ and $m_3 = 14k(4k+1)$;

(6°) G is a strongly regular graph of order n = 105(8k−3)^2 and degree r = 3(840k^2−630k+118) with τ = 945k^2−765k+153 and θ = 15(3k−1)(21k−8), where k ∈ N. Its eigenvalues are λ_2 = 15k−6 and λ_3 = −(105k−39) with $m_2 = 7(840k^2−630k+118)$ and $m_3 = 840k^2−630k+118$;

(7°) G is a strongly regular graph of order n = 105(8k−3)^2 and degree r = 5(840k^2−630k+118) with τ = 5(525k^2−387k+71) and θ = 15(5k−2)(35k−13), where k ∈ N. Its eigenvalues are λ_2 = 105k−40 and λ_3 = −(15k−5) with $m_2 = 840k^2−630k+118$ and $m_3 = 7(840k^2−630k+118)$;

(8°) G is a strongly regular graph of order n = 105(8k+3)^2 and degree r = 3(840k^2+630k+118) with τ = 945k^2+765k+153 and θ = 15(3k+1)(21k+153).
where $k \geq 0$. Its eigenvalues are $\lambda_2 = 105k+39$ and $\lambda_3 = -(15k+6)$ with $m_2 = 840k^2+630k+118$ and $m_3 = 7(840k^2+630k+118)$.

(50) $G$ is a strongly regular graph of order $n = 105(8k+3)^2$ and degree $r = 5(840k^2+630k+118)$ with $\tau = 5(525k^2+387k+71)$ and $\theta = 15(5k+2)(35k+13)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 15k+5$ and $\lambda_3 = -(105k+40)$ with $m_2 = 7(840k^2+630k+118)$ and $m_3 = 840k^2+630k+118$.

Proof. Firstly, according to Remark 2.4 we have $\alpha(\beta - 1) = 7(\alpha - 1)$, from which we find that $\alpha = 7$, $\beta = 7$. In view of this we obtain the strongly regular graph represented in Theorem 2.3 (10). Next, according to Proposition 2.5 it turns out that $G$ belongs to the class $(2^0)$ or $(3^0)$ or $(4^0)$ or $(5^0)$ if $m_2 = 7m_3$. According to Proposition 2.6 it turns out that $G$ belongs to the class $(2^0)$ or $(3^0)$ or $(4^0)$ or $(5^0)$ if $m_3 = 7m_2$. □

Proposition 2.7. Let $G$ be a connected strongly regular graph of order $n$ and degree $r$ with $m_2 = 8m_3$. Then $G$ belongs to the class $(4^0)$ or $(5^0)$ or $(6^0)$ or $(7^0)$ or $(8^0)$ or $(9^0)$ or $(10^0)$ or $(11^0)$ represented in Theorem 2.4.

Proof. Let $m_3 = p$, $m_2 = 8p$ and $n = 9p + 1$, where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(|\lambda_3| - 8\lambda_2)$. Let $|\lambda_3| - 8\lambda_2 = t$, where $t = 1, 2, \ldots, 8$. Let $\lambda_2 = k$, where $k$ is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_3 = -(8k + t)$; (ii) $\tau - \theta = -(7k + t)$; (iii) $\delta = 9k + t$; (iv) $r = pt$ and (v) $\theta = pt - 8k^2 - kt$. In this case we can easily see that (1.3) is reduced to

\[(p + 1)^2 - (9p + 1)t + 72k^2 + 16kt = 0.\] \hspace{1cm} (2.7)

Case 1. ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k + 1)$, $\tau - \theta = -(7k + 1)$, $\delta = 9k + 1$, $r = p$ and $\theta = p - 8k^2 - k$. Using (2.7) we find that $p = k(9k + 2)$. So we obtain that $G$ is a strongly regular graph of order $n = (9k + 1)^2$ and degree $r = k(9k + 2)$ with $\tau = k^2 - 6k - 1$ and $\theta = k(k + 1)$.

Case 2. ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k + 2)$, $\tau - \theta = -(7k + 2)$, $\delta = 9k + 2$, $r = 2p$ and $\theta = 2p - 8k^2 - 2k$. Using (2.7) we find that $7p - 1 = 4k(9k + 4)$. Replacing $k$ with $7k - 1$ we arrive at $p = 252k^2 - 56k + 3$. So we obtain that $G$ is a strongly regular graph of order $n = 28(9k - 1)^2$ and degree $r = 2(252k^2 - 56k + 3)$ with $\tau = 112k^2 - 63k + 5$ and $\theta = 14k(8k - 1)$.

Case 3. ($t = 3$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k + 3)$, $\tau - \theta = -(7k + 3)$, $\delta = 9k + 3$, $r = 3p$ and $\theta = 3p - 8k^2 - 3k$. Using (2.7) we find that $3p - 1 = 4k(3k + 2)$. Replacing $k$ with $3k + 1$ we arrive at $p = (2k + 1)(18k + 7)$. So we obtain that $G$ is a strongly regular graph of order $n = 4(9k + 4)^2$ and degree $r = 3(2k + 1)(18k + 7)$ with $\tau = 18k(2k + 1)$ and $\theta = (3k + 2)(12k + 5)$.

Case 4. ($t = 4$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k + 4)$, $\tau - \theta = -(7k + 4)$, $\delta = 9k + 4$, $r = 4p$ and $\theta = 4p - 8k^2 - 4k$. Using
(2.7) we find that $5p - 3 = 2k(9k + 8)$. Replacing $k$ with $5k - 1$ we arrive at $p = 90k^2 - 20k + 1$. So we obtain that $G$ is a strongly regular graph of order $n = 10(9k - 1)^2$ and degree $r = 4(90k^2 - 20k + 1)$ with $\tau = 160k^2 - 55k + 3$ and $\theta = 20k(8k - 1)$.

**Case 5.** $(t = 5)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k + 5)$, $\tau - \theta = -(7k + 5)$, $\delta = 9k + 5$, $r = 5p$ and $\theta = 5p - 8k^2 - 5k$. Using (2.7) we find that $5p - 5 = 2k(9k + 10)$. Replacing $k$ with $5k$ we arrive at $p = 90k^2 + 20k + 1$. So we obtain that $G$ is a strongly regular graph of order $n = 10(9k + 1)^2$ and degree $r = 5(90k^2 + 20k + 1)$ with $\tau = 10k(25k + 4)$ and $\theta = 5(5k + 1)(10k + 1)$.

**Case 6.** $(t = 6)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k + 6)$, $\tau - \theta = -(7k + 6)$, $\delta = 9k + 6$, $r = 6p$ and $\theta = 6p - 8k^2 - 6k$. Using (2.7) we find that $3p - 5 = 4k(3k + 4)$. Replacing $k$ with $3k - 2$ we arrive at $p = (2k - 1)(18k - 7)$. So we obtain that $G$ is a strongly regular graph of order $n = 4(9k - 4)^2$ and degree $r = 6(2k - 1)(18k - 7)$ with $\tau = 3(48k^2 - 45k + 10)$ and $\theta = 2(3k - 1)(24k - 11)$.

**Case 7.** $(t = 7)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k + 7)$, $\tau - \theta = -(7k + 7)$, $\delta = 9k + 7$, $r = 7p$ and $\theta = 7p - 8k^2 - 7k$. Using (2.7) we find that $7p - 21 = 4k(9k + 14)$. Replacing $k$ with $7k$ we arrive at $p = 252k^2 + 56k + 3$. So we obtain that $G$ is a strongly regular graph of order $n = 28(9k + 1)^2$ and degree $r = 7(252k^2 + 56k + 3)$ with $\tau = 14(7k + 1)(14k + 1)$ and $\theta = 7(7k + 1)(28k + 3)$.

**Case 8.** $(t = 8)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(8k + 8)$, $\tau - \theta = -(7k + 8)$, $\delta = 9k + 8$, $r = 8p$ and $\theta = 8p - 8k^2 - 8k$. Using (2.7) we find that $p = (k + 1)(9k + 7)$. Replacing $k$ with $k - 1$ we arrive at $p = k(9k - 2)$. So we obtain that $G$ is a strongly regular graph of order $n = (9k - 1)^2$ and degree $r = 8k(9k - 2)$ with $\tau = 64k^2 - 15k - 1$ and $\theta = 8k(8k - 1)$.

**Proposition 2.8.** Let $G$ be a connected strongly regular graph of order $n$ and degree $r$ with $m_3 = 8m_2$. Then $G$ belongs to the class $(4^0)$ or $(5^0)$ or $(6^0)$ or $(7^0)$ or $(8^0)$ or $(9^0)$ or $(10^0)$ or $(11^0)$ represented in Theorem 2.4.

**Proof.** Let $m_2 = p$, $m_3 = 8p$ and $n = 9p + 1$, where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(8|\lambda_3| - \lambda_2)$. Let $8|\lambda_3| - \lambda_2 = t$, where $t = 1, 2, \ldots, 8$. Let $\lambda_3 = -k$, where $k$ is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_2 = 8k - t$; (ii) $\tau - \theta = 7k - t$; (iii) $\delta = 9k - t$; (iv) $r = pt$ and (v) $\theta = pt - 8k^2 + kt$. In this case we can easily see that (1.4) is reduced to

\[(p + 1)t^2 - (9p + 1)t + 72k^2 - 16kt = 0. \quad (2.8)\]

**Case 1.** $(t = 1)$. Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 8k - 1$ and $\lambda_3 = -k$, $\tau - \theta = 7k - 1$, $\delta = 9k - 1$, $r = p$ and $\theta = p - 8k^2 + k$. Using (2.8) we find that $p = k(9k - 2)$. So we obtain that $G$ is a strongly regular graph of order $n = (9k - 1)^2$ and degree $r = k(9k - 2)$ with $\tau = k^2 + 6k - 1$ and $\theta = k(k - 1)$.\]
Case 2. \((t = 2)\). Using (i), (ii), (iii), (iv) and (v) we find that \(\lambda_2 = 8k - 2\) and \(\lambda_3 = -k\), \(\tau - \theta = 7k - 2\), \(\delta = 9k - 2\), \(r = 2p\) and \(\theta = 2p - 8k^2 + 2k\). Using (2.8) we find that \(7p - 1 = 4k(9k - 4)\). Replacing \(k\) with \(7k + 1\) we arrive at \(p = 252k^2 + 56k + 3\). So we obtain that \(G\) is a strongly regular graph of order \(n = 28(9k + 1)^2\) and degree \(r = 2(252k^2 + 56k + 3)\) with \(\tau = 112k^2 + 63k + 5\) and \(\theta = 14k(8k + 1)\).

Case 3. \((t = 3)\). Using (i), (ii), (iii), (iv) and (v) we find that \(\lambda_2 = 8k - 3\) and \(\lambda_3 = -k\), \(\tau - \theta = 7k - 3\), \(\delta = 9k - 3\), \(r = 3p\) and \(\theta = 3p - 8k^2 + 3k\). Using (2.8) we find that \(3p - 1 = 4k(3k - 2)\). Replacing \(k\) with \(3k - 1\) we arrive at \(p = (2k - 1)(18k - 7)\). So we obtain that \(G\) is a strongly regular graph of order \(n = 4(9k - 4)^2\) and degree \(r = 3(2k - 1)(18k - 7)\) with \(\tau = 18k(2k - 1)\) and \(\theta = (3k - 2)(12k - 5)\).

Case 4. \((t = 4)\). Using (i), (ii), (iii), (iv) and (v) we find that \(\lambda_2 = 8k - 4\) and \(\lambda_3 = -k\), \(\tau - \theta = 7k - 4\), \(\delta = 9k - 4\), \(r = 4p\) and \(\theta = 4p - 8k^2 + 4k\). Using (2.8) we find that \(5p - 3 = 2k(9k - 8)\). Replacing \(k\) with \(5k + 1\) we arrive at \(p = 90k^2 + 20k + 1\). So we obtain that \(G\) is a strongly regular graph of order \(n = 10(9k+1)^2\) and degree \(r = 4(90k^2 + 20k + 1)\) with \(\tau = 160k^2 + 55k + 3\) and \(\theta = 20k(8k + 1)\).

Case 5. \((t = 5)\). Using (i), (ii), (iii), (iv) and (v) we find that \(\lambda_2 = 8k - 5\) and \(\lambda_3 = -k\), \(\tau - \theta = 7k - 5\), \(\delta = 9k - 5\), \(r = 5p\) and \(\theta = 5p - 8k^2 + 5k\). Using (2.8) we find that \(5p - 5 = 2k(9k - 10)\). Replacing \(k\) with \(5k\) we arrive at \(p = 90k^2 - 20k + 1\). So we obtain that \(G\) is a strongly regular graph of order \(n = 10(9k - 1)^2\) and degree \(r = 5(90k^2 - 20k + 1)\) with \(\tau = 10k(25k - 4)\) and \(\theta = 5(5k - 1)(10k - 1)\).

Case 6. \((t = 6)\). Using (i), (ii), (iii), (iv) and (v) we find that \(\lambda_2 = 8k - 6\) and \(\lambda_3 = -k\), \(\tau - \theta = 7k - 6\), \(\delta = 9k - 6\), \(r = 6p\) and \(\theta = 6p - 8k^2 + 6k\). Using (2.8) we find that \(3p - 5 = 4k(3k - 4)\). Replacing \(k\) with \(3k + 2\) we arrive at \(p = (2k + 1)(18k + 7)\). So we obtain that \(G\) is a strongly regular graph of order \(n = 4(9k + 4)^2\) and degree \(r = 6(2k + 1)(18k + 7)\) with \(\tau = 3(48k^2 + 45k + 10)\) and \(\theta = 2(3k + 1)(24k + 11)\).

Case 7. \((t = 7)\). Using (i), (ii), (iii), (iv) and (v) we find that \(\lambda_2 = 8k - 7\) and \(\lambda_3 = -k\), \(\tau - \theta = 7k - 7\), \(\delta = 9k - 7\), \(r = 7p\) and \(\theta = 7p - 8k^2 + 7k\). Using (2.8) we find that \(7p - 21 = 4k(9k - 14)\). Replacing \(k\) with \(7k\) we arrive at \(p = 252k^2 - 56k + 3\). So we obtain that \(G\) is a strongly regular graph of order \(n = 28(9k - 1)^2\) and degree \(r = 7(252k^2 - 56k + 3)\) with \(\tau = 14(7k - 1)(14k - 1)\) and \(\theta = 7(7k - 1)(28k - 3)\).

Case 8. \((t = 8)\). Using (i), (ii), (iii), (iv) and (v) we find that \(\lambda_2 = 8k - 8\) and \(\lambda_3 = -k\), \(\tau - \theta = 7k - 8\), \(\delta = 9k - 8\), \(r = 8p\) and \(\theta = 8p - 8k^2 + 8k\). Using (2.8) we find that \(p = (k - 1)(9k - 7)\). Replacing \(k\) with \(k + 1\) we arrive at \(p = k(9k + 2)\). So we obtain that \(G\) is a strongly regular graph of order \(n = (9k + 1)^2\) and degree \(r = 8k(9k + 2)\) with \(\tau = 64k^2 + 15k - 1\) and \(\theta = 8k(8k + 1)\). □

Remark 2.10. We note that the complete bipartite graph \(K_{5,5}\) is a strongly regular graph with \(m_2 = 8m_3\). It is obtained from the class Theorem 2.4 \((\overline{G})^0\) for \(k = 0\).

Remark 2.11. We note that \(\overline{K}_7\) is a strongly regular graph with \(m_2 = 8m_3\). It is obtained from the class Theorem 2.4 \((TT)^0\) for \(k = 0\).
Remark 2.12. We note that $8K_5$ is a strongly regular graph with $m_2 = 8m_3$. It is obtained from the class Theorem 2.4 $(\mathcal{I}^0)$ for $k = 1$.

**Theorem 2.4.** Let $G$ be a connected strongly regular graph of order $n$ and degree $r$ with $m_2 = 8m_3$ or $m_3 = 8m_2$. Then $G$ is one of the following strongly regular graphs:

1. $G$ is the complete bipartite graph $K_{5,5}$ of order $n = 10$ and degree $r = 5$ with $\tau = 0$ and $\theta = 5$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -5$ with $m_2 = 8$ and $m_3 = 1$;
2. $G$ is the strongly regular graph $4K_7$ of order $n = 28$ and degree $r = 21$ with $\tau = 14$ and $\theta = 21$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -7$ with $m_2 = 24$ and $m_3 = 3$;
3. $G$ is the strongly regular graph $8K_8$ of order $n = 64$ and degree $r = 56$ with $\tau = 48$ and $\theta = 56$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -8$ with $m_2 = 56$ and $m_3 = 7$;
4. $G$ is a strongly regular graph of order $n = (9k-1)^2$ and degree $r = k(9k-2)$ with $\tau = k^2 + 6k - 1$ and $\theta = k(k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 8k - 1$ and $\lambda_3 = -k$ with $m_2 = k(9k-2)$ and $m_3 = 8k(9k-2)$;
5. $G$ is a strongly regular graph of order $n = (9k+1)^2$ and degree $r = k(9k+2)$ with $\tau = k^2 - 6k - 1$ and $\theta = k(k+1)$, where $k \geq 7$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(8k+1)$ with $m_2 = 8k(9k+2)$ and $m_3 = k(9k+2)$;
6. $G$ is a strongly regular graph of order $n = 4(9k-4)^2$ and degree $r = 3(2k-1)(18k-7)$ with $\tau = 18k(2k-1)$ and $\theta = (3k-2)(12k-5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 24k-11$ and $\lambda_3 = -(3k-1)$ with $m_2 = (2k-1)(18k-7)$ and $m_3 = 8(2k-1)(18k-7)$;
7. $G$ is a strongly regular graph of order $n = 4(9k-4)^2$ and degree $r = 6(2k-1)(18k-7)$ with $\tau = 3(48k^2 - 45k + 10)$ and $\theta = 2(3k-1)(24k-11)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k - 2$ and $\lambda_3 = -(24k - 10)$ with $m_2 = 8(2k-1)(18k-7)$ and $m_3 = (2k-1)(18k-7)$;
8. $G$ is a strongly regular graph of order $n = 4(9k+4)^2$ and degree $r = 3(2k+1)(18k+7)$ with $\tau = 18k(2k+1)$ and $\theta = (3k+2)(12k+5)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 3k+1$ and $\lambda_3 = -(24k+11)$ with $m_2 = 8(2k+1)(18k+7)$ and $m_3 = (2k+1)(18k+7)$;
9. $G$ is a strongly regular graph of order $n = 4(9k+4)^2$ and degree $r = 6(2k+1)(18k+7)$ with $\tau = 3(48k^2 + 45k + 10)$ and $\theta = 2(3k+1)(24k+11)$, where
\[ k \geq 0. \] Its eigenvalues are \( \lambda_2 = 24k+10 \) and \( \lambda_3 = -(3k+2) \) with \( m_2 = (2k+1)(18k+7) \) and \( m_3 = 8(2k+1)(18k+7) \);

\((8^0)\) \( G \) is a strongly regular graph of order \( n = 10(9k-1)^2 \) and degree \( r = 4(90k^2-20k+1) \) with \( \tau = 160k^2-55k+3 \) and \( \theta = 20k(8k-1) \), where \( k \in \mathbb{N} \). Its eigenvalues are \( \lambda_2 = 5k-1 \) and \( \lambda_3 = -(40k-4) \) with \( m_2 = 8(90k^2-20k+1) \) and \( m_3 = 90k^2-20k+1 \);

\((8^0)\) \( G \) is a strongly regular graph of order \( n = 10(9k-1)^2 \) and degree \( r = 4(90k^2-20k+1) \) with \( \tau = 160k^2-55k+3 \) and \( \theta = 20k(8k+1) \), where \( k \in \mathbb{N} \). Its eigenvalues are \( \lambda_2 = 40k-5 \) and \( \lambda_3 = -5k \) with \( m_2 = 90k^2-20k+1 \) and \( m_3 = 8(90k^2-20k+1) \);

\((9^0)\) \( G \) is a strongly regular graph of order \( n = 10(9k+1)^2 \) and degree \( r = 4(90k^2+20k+1) \) with \( \tau = 10k(25k+4) \) and \( \theta = 5(5k-1)(10k+1) \), where \( k \in \mathbb{N} \). Its eigenvalues are \( \lambda_2 = 40k+4 \) and \( \lambda_3 = -(5k+1) \) with \( m_2 = 90k^2+20k+1 \) and \( m_3 = 8(90k^2+20k+1) \);

\((9^0)\) \( G \) is a strongly regular graph of order \( n = 10(9k+1)^2 \) and degree \( r = 4(90k^2+20k+1) \) with \( \tau = 10k(25k+4) \) and \( \theta = 5(5k+1)(10k+1) \), where \( k \in \mathbb{N} \). Its eigenvalues are \( \lambda_2 = 5k \) and \( \lambda_3 = -(40k+5) \) with \( m_2 = 8(90k^2+20k+1) \) and \( m_3 = 90k^2+20k+1 \);

\((10^0)\) \( G \) is a strongly regular graph of order \( n = 28(9k-1)^2 \) and degree \( r = 2(252k^2-56k+3) \) with \( \tau = 112k^2-63k+5 \) and \( \theta = 14k(8k-1) \), where \( k \in \mathbb{N} \). Its eigenvalues are \( \lambda_2 = 7k-1 \) and \( \lambda_3 = -(56k-6) \) with \( m_2 = 8(252k^2-56k+3) \) and \( m_3 = 252k^2-56k+3 \);

\((10^0)\) \( G \) is a strongly regular graph of order \( n = 28(9k+1)^2 \) and degree \( r = 7(252k^2-56k+3) \) with \( \tau = 14(7k-1)(14k-1) \) and \( \theta = 7(7k-1)(28k-3) \), where \( k \in \mathbb{N} \). Its eigenvalues are \( \lambda_2 = 56k-7 \) and \( \lambda_3 = -7k \) with \( m_2 = 252k^2-56k+3 \) and \( m_3 = 8(252k^2-56k+3) \);

\((11^0)\) \( G \) is a strongly regular graph of order \( n = 28(9k+1)^2 \) and degree \( r = 2(252k^2+56k+3) \) with \( \tau = 112k^2+63k+5 \) and \( \theta = 14k(8k+1) \), where \( k \in \mathbb{N} \). Its eigenvalues are \( \lambda_2 = 56k+6 \) and \( \lambda_3 = -(7k+1) \) with \( m_2 = 252k^2+56k+3 \) and \( m_3 = 8(252k^2+56k+3) \);

\((11^0)\) \( G \) is a strongly regular graph of order \( n = 28(9k+1)^2 \) and degree \( r = 7(252k^2+56k+3) \) with \( \tau = 14(7k+1)(14k+1) \) and \( \theta = 7(7k+1)(28k+3) \), where \( k \in \mathbb{N} \). Its eigenvalues are \( \lambda_2 = 7k \) and \( \lambda_3 = -(56k+7) \) with \( m_2 = 8(252k^2+56k+3) \) and \( m_3 = 252k^2+56k+3 \).

Proof. Firstly, according\(^2\) to Remark 2.4 we have \( \alpha(\beta - 1) = 8(\alpha - 1) \), from which we find that \( \alpha = 2, \beta = 5 \) or \( \alpha = 4, \beta = 7 \) or \( \alpha = 8, \beta = 8 \). In view of this\(^3\) we

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\(^2\)The all results presented in this work are verified by using a computer program srgpar.exe, which has been written by the author in the programming language Borland C++ Builder 5.5.

\(^3\)One can use the web page https://www.win.tue.nl/~aeb/graphs/srg/srgtab.html that contains the parameters of strongly regular graphs from 5 upto 1300 vertices.
obtain the strongly regular graphs represented in Theorem 2.4 (1°), (2°), (3°).
Next, according to Proposition 2.7 it turns out that $G$ belongs to the class (4°) or (5°) or (6°) or (7°) or (8°) or (10°) or (11°) if $m_2 = 8m_3$. According to Proposition 2.8 it turns out that $G$ belongs to the class (4°) or (5°) or (6°) or (7°) or (10°) or (11°) if $m_3 = 8m_2$.

□

Remark 2.13. We note that for some values of the parameter $k$ it is possible that there exist no strongly regular graph. In view of this, we in this work describe the all feasible parameters $n, r, \tau$ and $\Theta$ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 5, 6, 7, 8$.

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